PHY3011 Quantum Mechanics—Assignment 3

Please do this assignment and hand in the answers to the Departmental Office by 4pm Monday 10th December.

I will only award full marks for answers that are neatly presented, legible and correct. If you tend to make a mess when you do algebra, then make a fair copy when you are finished and hand that in.

1. Here are four equations involving orbital angular momentum,

$$\hat{L}^2|\ell\, m\rangle = \hbar^2\ell\,(\ell+1)|\ell\, m\rangle \tag{1}$$
$$\hat{L}_z|\ell\, m\rangle = \hbar m|\ell\, m\rangle \tag{2}$$
$$\hat{L}_+|\ell\, m\rangle = C|\ell\;\; m+1\rangle \tag{3}$$
$$\hat{L}_-|\ell\, m\rangle = D|\ell\;\; m-1\rangle \tag{4}$$

(a) Which of these is an eigenvalue–eigenvector equation? Explain why.

(b) The operators in (3) and (4) are

$$\hat{L}_+ = \hat{L}_x + i\hat{L}_y$$
$$\hat{L}_- = \hat{L}_x - i\hat{L}_y$$

Use these as well as (1) and (2) to find the constants C and D in (3) and (4), showing all of your working and if necessary explaining in words what you are doing. **I will only award full marks if it is clear from your answer that you understand what you've done. It's up to you to persuade me.**

2. This is a variation on example 4.2 from Griffiths. Suppose a spin-$\frac{1}{2}$ particle is in the state

$$\chi = A\begin{pmatrix} 1-i \\ 2 \end{pmatrix}$$

(a) Find the constant A that normalises χ.

(b) What are the *expectation* (average) values of the spin angular momentum if you measure S_x, S_y or S_z? Do it using the Pauli matrices.

(c) Suppose you measure S_z and then immediately afterwards you measure S_y on this particle. In the second measurement, what is the probability that the outcome will be $\frac{1}{2}\hbar$? No calculation needed, but don't just *guess* either; explain briefly your reasoning.

../

3. The neutral K-meson decays into a pi meson, an antimuon and a muon neutrino,

$$K^0 \longrightarrow \pi^- + \mu^+ + \nu_\mu$$

and the antimuon emerging is always left handed, that is, in the state "spin down" if you like; and the neutrino is left handed which is what they all are. This conserves spin since the K^0 and π^- are spin-0 and the μ^+ and neutrino are spin-$\frac{1}{2}$ and fly off into opposite directions. The μ^+ has a proper lifetime of 2.2 μs after which it decays into a positron and two neutrinos,

$$\mu^+ \longrightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

In an experiment in 1973, Sandweiss *et al.* brought antimuons emerging from K^0 decay to rest in a block of aluminium and caused them to precess in, say, the xy-plane, by applying a uniform **B** field of 60 gauss in the z-direction. The probability for positron emission is greatest in a direction parallel to the antimuon's spin, so it acts like a lighthouse emitting a beam whose light is rotating in the xy plane at the antimuon's precession frequency. Sandweiss *et al.* found this to be 807.5 kHz. Using these data calculate the gyromagnetic ratio of the antimuon, expressing your answer in units of (rad s^{-1} T^{-1}). Compare with the data in your notes.

Since the antimuon only emits *one* positron before it decays, how can this lighthouse picture possibly apply to this experiment? (Remember Einstein's words…)

[1 Gauss is 10^{-4} Tesla]